Gravitational waveforms in scalar-tensor theories

Laura Bernard (LUTH - Observatoire de Paris)

Athens 2019

Gravitational Waves, Black Holes and Fundamental Physics

How to modify General Relativity ?

The scalar-tensor action

$$S_{\mathrm{ST}} = \frac{c^3}{16\pi G}\int \mathrm{d}^4x\sqrt{-g}\left[\phi R - \frac{\omega(\phi)}{\phi} g^{\alpha\beta}\partial_\alpha\partial_\beta\phi\right] + S_m\left(\mathfrak{m}, g_{\alpha\beta}\right)$$

- **Simple**: add one massless scalar field
- **Minimal**: scalar field coupled only to gravity, not to the matter
  - Weak-field tests: Solar System, binary pulsar tests
  - Strong constraints on the parameters of the theory

ST action in Einstein frame

Conformal transformation

$$\tilde{g}_{\mu\nu} = \varphi \, g_{\mu\nu} \,, \qquad \varphi = \frac{\phi}{\phi_0} \qquad \text{with } \phi_0 = \phi(\infty) = \text{cst}$$

$$S_{\mathrm{ST}} = \frac{c^3 \, \phi_0}{16\pi G} \int \mathrm{d}^4 x \sqrt{-\tilde{g}} \left[\tilde{R} - \frac{3 + 2\omega(\phi)}{2\varphi^2} \tilde{g}^{\alpha\beta} \partial_\alpha \varphi \partial_\beta \varphi \right] + S_m \left(\mathfrak{m}, \frac{\tilde{g}_{\alpha\beta}}{\varphi} \right)$$

- BHs indistinguishable from GR
- **Still interesting**: Strong deviations from GR for neutron stars
- **Well-posed** initial value problem

Violation of the Strong Equivalence Principle

- Incorporate the internal structure of compact, self-gravitating bodies
- **Eardley's approach**: masses depend on the scalar field $m_A(\phi)$

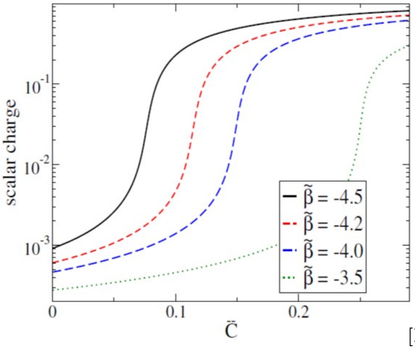
$$S_\mathrm{m} = -\sum_A \int \mathrm{d}t\, m_A(\phi)\, c^2 \sqrt{-g_{\alpha\beta}\frac{v_A^\alpha v_A^\beta}{c^2}}$$

- **Sensitivities**: $s_A = \left.\frac{\mathrm{d}\ln m_A(\phi)}{\mathrm{d}\ln\phi}\right|_0$
  - Neutron stars: $s_A \sim 0.2$ (depends on the equation of states)
  - Black holes: $s_A = 0.5$ (compacity M/R)
  - related to the scalar charge $\alpha_A \propto 1 - 2s_A$
- Higher order: $\tilde{\beta} \propto \left.\frac{\mathrm{d}^2\ln m_A(\phi)}{\mathrm{d}\ln\phi^2}\right|_0$

Binary coalescence in ST theories

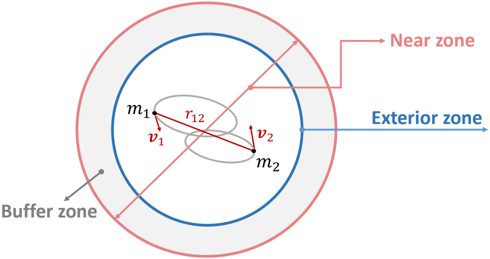
Late inspiral - merger

- ▷ Spontaneous scalarisation
- ▷ Dynamical scalarisation
  - Numerical results
  - Analytical approach using resummation tecniques



[Palenzuela et al, 2014]

Isolated, compact, **slowly moving** and **weakly stressed** source



$$\epsilon \equiv \frac{v_{12}^2}{c^2} \sim \frac{Gm}{r_{12}c^2} \ll 1$$

Method of asymptotic matching

- Near zone : post-Newtonian expansion, $n\text{PN} = \mathcal{O}\left(\frac{1}{c^{2n}}\right)$
- Exterior zone : multipolar expansion in power of $\frac{r_{12}}{R}$

▶ **radiative moments** $\underset{\text{exp. in } 1/R}{\longleftarrow}$ **source moments** $\underset{\text{matching}}{\longrightarrow}$ **source**

Binary coalescence in ST theories

Inspiral - towards 2PN waveforms

- Equations of motion at 3PN
- Tensor gravitational waveform to 2PN
- Scalar waveform to 1.5PN: **starts at −0.5PN**
- Energy flux to 1PN: **starts at −1PN**

$$\frac{\mathrm{d}E_{\mathrm{dipole}}}{\mathrm{d}t} = \frac{4m\nu^2}{3rc^3}\left(\frac{G_{\mathrm{eff}}m}{r}\right)^3(\alpha_2-\alpha_1)^2$$

Binary coalescence in ST theories

Inspiral - towards 2PN waveforms

- Equations of motion at 3PN (mPN formalism)
- Tensor gravitational waveform to 2PN (DIRE formalism)
- Scalar waveform to 1.5PN: **starts at −0.5PN** (DIRE)
- Energy flux to 1PN: **starts at −1PN** (DIRE)

$$\frac{\mathrm{d}E_{\text{dipole}}}{\mathrm{d}t} = \frac{4m\nu^2}{3rc^3}\left(\frac{G_{\text{eff}}m}{r}\right)^3(\alpha_2-\alpha_1)^2$$

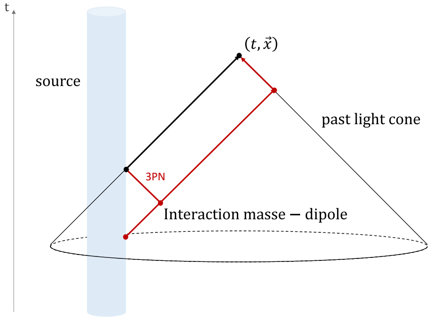
Still to do

- Potential issues when matching:
  - Same harmonic gauge?
  - Regularisation procedure $\longrightarrow$ dimensional regularisation in both cases ?

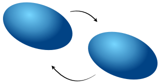
3PN equations of motion

$$\frac{\mathrm{d}\mathbf{v_1}}{\mathrm{d}t} = \underbrace{-\frac{G_{\text{eff}}\, m_2}{r_{12}^2}\mathbf{n_{12}} + \frac{\mathbf{A}_{1\text{PN}}}{c^2}}_{\text{conservative terms}} + \underbrace{\frac{\mathbf{A}_{1.5\text{PN}}}{c^3}}_{\text{rad. reac.}} + \underbrace{\frac{\mathbf{A}_{2\text{PN}}}{c^4}}_{\text{cons.}} + \underbrace{\frac{\mathbf{A}_{2.5\text{PN}}}{c^5}}_{\text{rad. reac.}} + \underbrace{\frac{\mathbf{A}^{\text{inst}}_{3\text{PN}}}{c^6}}_{\text{cons, local}} + \underbrace{\frac{\mathbf{A}^{\text{tail}}_{3\text{PN}}}{c^6}}_{\text{cons, nonloc.}}$$

▷ **A scalar tail term: new ST effect at 3PN**



$$L_{\text{tail}} \propto \frac{1}{c^6} I_i^{(2)}(t) \int_0^{+\infty} \mathrm{d}t\, \ln\left(\frac{\tau}{\tau_0}\right) I_i^{(3)}(t-\tau)$$



Reminder - General relativity

$$S_{\text{extended bodies}} = S_{\text{p.p.}} + \int \left(k_2 C^2_{\mu\nu\rho\sigma} + k_4 C^2 u^2 + \cdots \right) c \, \mathrm{d}s$$

- $\mathcal{Q}_{\mu\nu} = -\lambda_2 \, \mathcal{E}_{\mu\nu}$
- starts at 5PN.
- Electric and magnetic-type tidal Love number: k_l^{E} and k_l^{B},

The scalar tidal effect

$$\Delta S_{(fs)} = -\frac{1}{2}\sum_A \lambda_A^{(s)} \int \mathrm{d}s_A \, \left(g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi\right)_A$$

- Time-varying scalar dipole moment $\propto$ external tidal field

$$\mathcal{Q}_\mu^{(s)} = -\lambda_{(s)}\mathcal{E}_\mu^{(s)}$$

- Scalar-type Love number $\lambda_{(s)}$

Consequence on scalar radiation

- 3PN order in the dynamics $\Delta\mathbf{a}_{(fs)} \sim \mathbf{a}_{(\mathrm{N})} \cdot \tilde{\lambda}^{(s)} \, x^3$ $\qquad x = (m\omega)^{2/3}$
- small ST parameters but scales as $\left(\frac{R}{M}\right)^3$
- $\Delta\psi_{(fs)} \sim -\frac{1}{4\zeta(\alpha_1-\alpha_2)^2\eta x^{3/2}} \, \tilde{\Lambda}_{(s)} \, x^3 \Longrightarrow$ 2PN effect in the phase

Towards a full IMR waveform

- PN inspiral modelling
- Tidal effects - scalar-type Love number
- Effective one body formalism
- Late-inspiral effects
- Matching to numerical relativity