Hearing the Nature of Compact Objects

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Theoretical Astropyhsics

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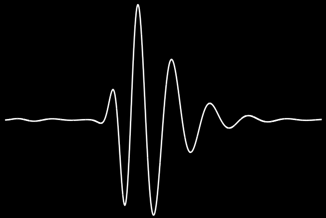
Gravitational Waves, Black Holes and Fundamental Physics

21-24 January 2019

Athens, Greece



PART I

Compact Objects and the Horizon Scale

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GW Echoes $\Leftrightarrow$ Exotic Horizon Scale Physics?

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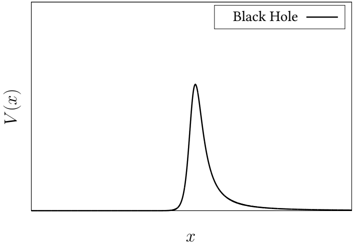

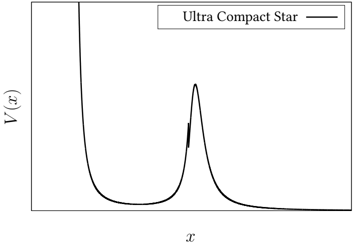

GRAVITATIONAL WAVE ECHOES: PHENOMENOLOGY

- Simplest scenario: spherically symmetric and non-rotating objects

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Kokkotas and Schmidt (1999); Nollert (1999); Berti et al. (2009) Maggiore (2018)

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- $V(r) \Rightarrow \omega_n^2$ characteristic for the object[2]

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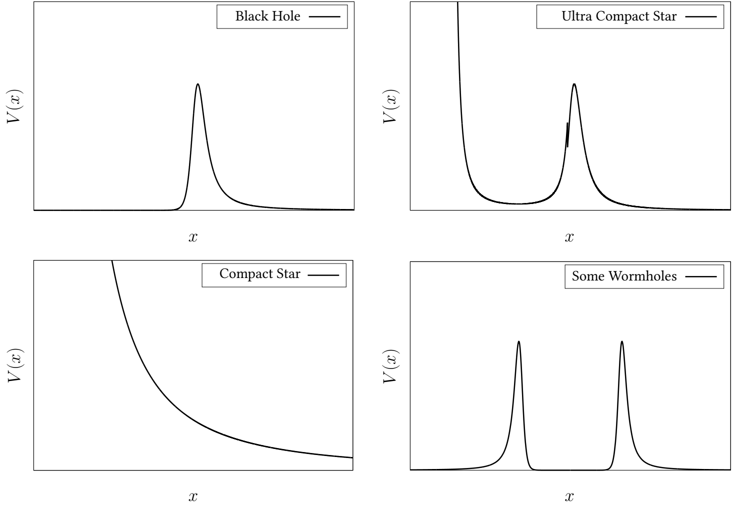
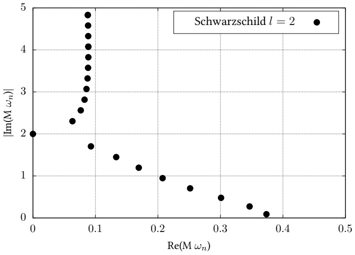
Perturbation Potentials of Compact Objects



QNM SPECTRA OF COMPACT OBJECTS



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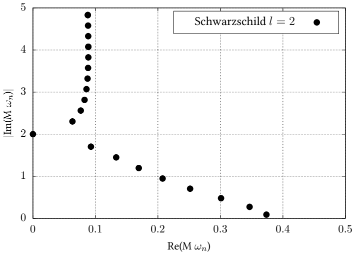
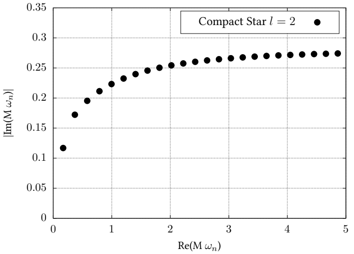
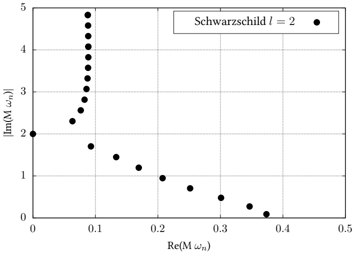
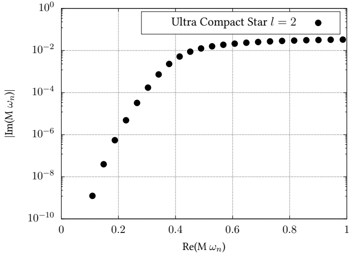
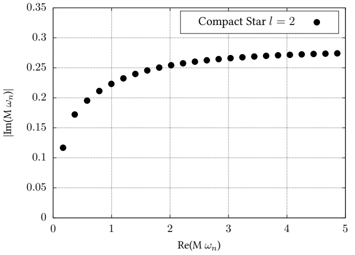




QNM spectra of Compact Objects







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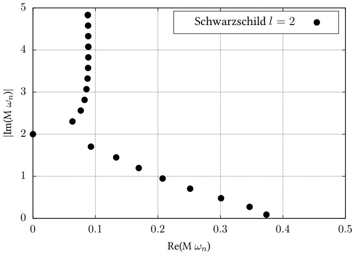

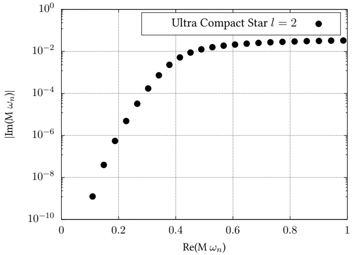

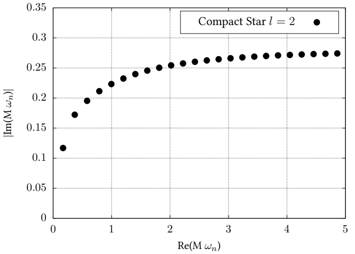

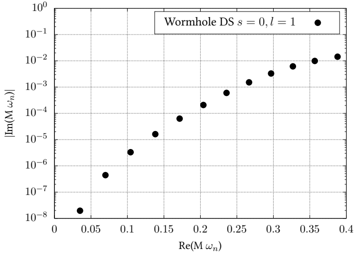

PART II

Mode Calculations and Inverse Problem

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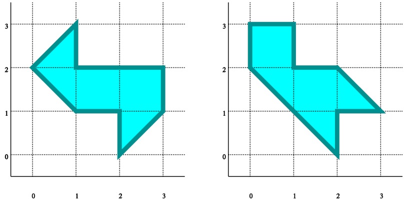
Can one hear the shape of a drum?[3]



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WKB method and Bohr-Sommerfeld Rules

- Methods for **direct** QNM calculations:
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$$\int_{x_0}^{x_1} \sqrt{E_n - V(x)}\mathrm{d}x = \pi\left(n + \frac{1}{2}\right) - \frac{i}{4}\exp\left(2i\int_{x_1}^{x_2}\sqrt{E_n - V(x)}\mathrm{d}x\right) \tag{2}$$

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INVERT BS RULES

- Known for **single** wells and **single** barriers (**classical BS rule)**[5]

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$$\mathcal{L}_1(E) = x_1 - x_0 = 2\frac{\partial}{\partial E}\int_{E_{\text{min}}}^{E} \frac{n(E') + 1/2}{\sqrt{E - E'}} \mathbf{d}E' \tag{3}$$

$$\mathcal{L}_2(E) = x_2 - x_1 = -\frac{1}{\pi}\int_{E}^{E_{\text{max}}} \frac{(\mathbf{d}T(E')/\mathbf{d}E')}{T(E')\sqrt{E' - E}} \mathbf{d}E' \tag{4}$$

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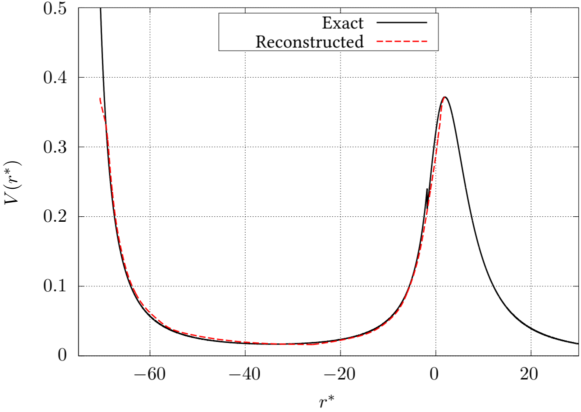
Ultra Compact Stars

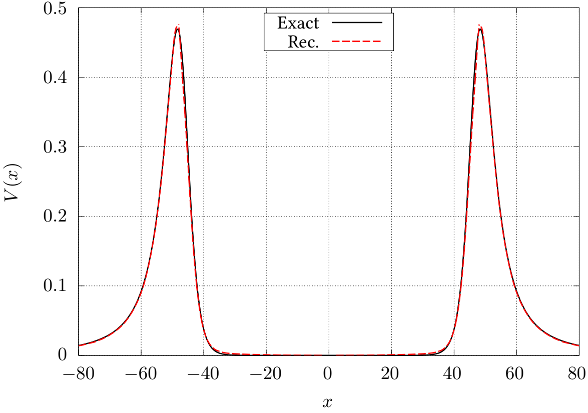
Example for ultra compact constant density star $C \approx 0.44$



Reconstructed axial perturbation potential, constant density star, $l = 3$, taken from Völkel and Kokkotas (2017,2).

Damour-Solodukhin Wormhole

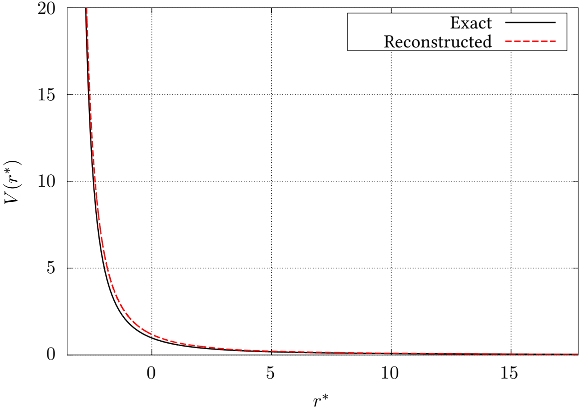
Example for Damour-Solodukhin wormhole $C \approx 0.5$



Reconstructed scalar perturbation potential, Damour-Solodukhin wormhole, $l = 3$, taken from Völkel and Kokkotas (2018,2).

NEUTRON STARS

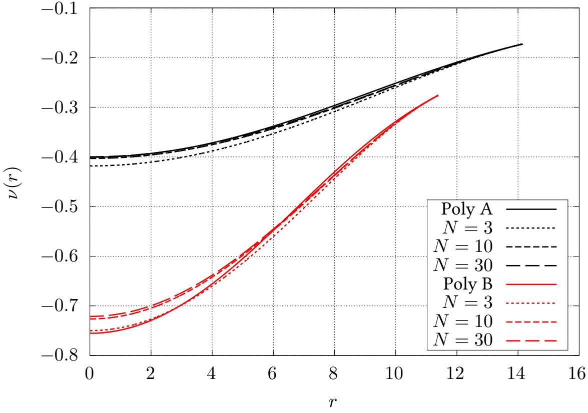
Example for neutron star polytrope $C \approx 0.15$



Reconstructed axial perturbation potential, neutron star polytrope, $l = 3$, Völkel and Kokkotas (2019 TBS).

Neutron Stars

Fit spectrum to analytic model potential for neutron star
Bohr-Sommerfeld rule to reconstruct internal space-time



Reconstructed metric function $\nu(r)$, for two neutron star polytropes, $l = 3$, Völkel and Kokkotas (2019 TBS).

PART III

Significance of Eigenvalues/QNMs

Eigenvalue Spectrum vs. Waveform

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- How strong **eigenvalues** are **excited** depend on the **initial perturbation**

SIGNIFICANCE OF EIGENVALUES/QNMS

Be careful:

- **"Discrepancy"** with respect to **time-evolution** possible (Scattering with black hole barrier + reflection close to horizon Cardoso et al. (2016); Abedi et al. (2017))

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- **Discontinuities** in potential may change spectrum, but only negligible time-evolution (Nollert (1996); Nollert and Price (1999) Schwarzschild step potential)
- “**Environmental effects**” can change QNM spectrum, but might only slightly modify time-evolution (Barausse et al. (2014); Konoplya et al. (2019))

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- **Reconstruction** of **perturbation potential** can be possible
- **Potentials** imprint **space-time** and **equation of state**
- **Exotic objects** templates are "**more involved**" than black holes/neutron stars (**unknown physics**)

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